

Criteria for Infinite Avalanches in Zero-Temperature Nonequilibrium Random-Field Ising Model on a Bethe Lattice

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We present general criteria for the occurrence of infinite avalanches and critical hysteresis in the zero-temperature nonequilibrium random-field Ising model on a Bethe lattice. Drawing upon extant results as well as a new result on a dilute four-coordinated ($z = 4$) lattice, we show that diverging avalanches can occur if an arbitrarily small fraction of sites on a spanning cluster have connectivity $z \geq 4$.

I. INTRODUCTION

Zero-temperature nonequilibrium random-field Ising model has been used extensively to understand hysteresis in systems with quenched disorder [1–8]. Analysis and simulations of the model reveal that character of hysteresis loop depends upon several factors including probability distribution of the random-field, dimensionality and connectivity of the lattice. A common choice for the random-field distribution is a Gaussian with average zero and standard deviation σ . In some cases depending upon the dimensionality and connectivity of the lattice, there exists a critical value $\sigma = \sigma_c$ such that each half of the hysteresis loop has a critical point characterized by diverging susceptibility of the system. For $\sigma < \sigma_c$, there is a discontinuity in each half of the loop arising from a massive flipping of spins triggered by an infinitesimal change in the applied field, i.e. an infinite avalanche. Infinite avalanches and a critical point go hand in hand in the present model. Infinite avalanches end in a critical point as σ is increased to its critical value. While the critical behavior in the vicinity of σ_c has been investigated extensively, the criteria for the existence of σ_c , its dependence on the dimensionality and connectivity of the lattice has received less attention. As discussed below, general conditions for the occurrence of infinite avalanches in this simple model remain unclear so far. This brief report addresses this issue drawing upon extant results as well as a new result presented here.

II. THE MODEL, EXTANT RESULTS, AND RELATED ISSUES

We begin by describing the model briefly, listing known results and related issues. The Hamiltonian of the model is,

$$H = -J \sum_{i,j} s_i s_j - h \sum_i s_i - \sum_i h_i s_i$$

Here $s_i = \pm 1$ is an Ising spin at site i , h is a uniform applied field, and h_i is a quenched field drawn from a Gaussian distribution of mean zero and standard deviation σ ; J is ferromagnetic coupling between nearest neighbors on a lattice. The applied field h is assumed to vary infinitely slowly, and the dynamics of the model is taken to be the adiabatic zero-temperature single-spin-flip Glauber dynamics. At each value of h , spins in the system are flipped as needed till each spin s_i is aligned along the net local field ℓ_i at its site; $\ell_i = J \sum_{j \neq i} s_j + h + h_i$. We start at $h = -\infty$ with the stable state of the system having all spins down, and the magnetization per site $m(h) = -1$. Now h is increased till some spin flips up, say at $h = \tilde{h}$. A spin that flips up increases the field on its neighbors by $2J$, and some of them may flip up and so on. Thus a spin flipping up may initiate an avalanche of flipped up spins. Sites in an avalanche lie on a connected cluster whose size equals the size of the avalanche. Each avalanche of size s increases $m(h)$ by an amount $2s/N$ where N is the size of the system. When an avalanche is finished, the field h is increased again till the next avalanche occurs. This process is continued till all spins are up. As h is ramped up from $h = -\infty$ to $h = \infty$, $m(h)$ increases in tiny irregular steps separated by random quiescent intervals along the applied field. This is Barkhausen noise but not the main concern of the present paper. Our concern is with a discontinuity in $m(h)$ in the thermodynamic limit, i.e. a macroscopic avalanche of the order of N and the criteria for its occurrence.

What we know so far is that if a macroscopic avalanche occurs, it occurs for $\sigma < \sigma_c$ where σ_c is a critical value. If there is no σ_c , there is no avalanche. Also, there is only one avalanche, say at $h = \tilde{h}$. As $\sigma \rightarrow \sigma_c$, \tilde{h} decreases and so does the size of the avalanche. The size goes to zero at $\sigma = \sigma_c$, $h = \tilde{h}_c$ but fluctuations at this point are anomalously large. This is a nonequilibrium critical point with behavior similar to that of an equilibrium Ising model at its critical temperature T_c . Indeed, σ_c plays a role analogous to T_c . It is not clear why this should be so because

T_c takes into account thermal relaxation of all states of the system, but σ_c is based only on one initial state (all spins down) and its zero-temperature Glauber dynamics. The question we ask is whether T_c and σ_c are determined by similar criteria. Our minimal model is characterized by a small set of parameters: J, h, σ , and two implicit parameters d and z denoting the dimension and coordination number of the lattice. J sets the energy scale, h and σ are used as tuning parameters to locate a critical point if it exists. This means that the existence of a critical point (σ_c, \tilde{h}_c) must depend on d , or z , or both. For thermal model, the existence of T_c is decided by d alone which should be above the lower critical dimension d_ℓ for the system; $d_\ell = 1$ for the pure Ising model, $d_\ell = 2$ for the random-field Ising model [9]. For $d > d_\ell$, the temperature-driven critical behavior does not depend on z . For example, if there is a T_c on a square lattice ($d = 2, z = 4$), there is also a T_c on the honeycomb lattice ($d = 2, z = 3$). One may expect the same for the disorder-driven critical behavior at $T = 0$ on grounds that both types of critical phenomena are caused by a diverging length; diverging correlation length in one case and a diverging avalanche in the other. However, there is a σ_c on the square lattice [8] and the triangular lattice [10] but no σ_c on the honeycomb lattice [11]. We would like to understand why?

The coordination number $z = 4$ has a special connection with σ_c . The square lattice ($z = 4$) is commonly used for studying the behavior of a model in $d = 2$. Numerical efforts to find σ_c for the square lattice were inconclusive initially. This was thought to have a bearing on d_ℓ for the random-field Ising model which was in question initially. Eventually theory settled $d_\ell = 2$, and numerical work on large systems indicated the presence of σ_c on the square lattice. However, as mentioned above there appear to be other factors beside d_ℓ that determine σ_c . An analytic solution of the model on a Bethe lattice of integer coordination number z shows that infinite avalanches and critical phenomena occur only if $z \geq 4$ [12]. This is surprising because in all other cases, as far as we know, the critical behavior of an Ising model on a Bethe lattice does not depend upon z if $z > 2$. Evidently, there is not a very simple and clear physical reason for the absence of infinite avalanches on a $z = 3$ Bethe lattice. It has been explained by mapping the problem to a branching process in population dynamics [13]. The unusual dependence of σ_c on the coordination number z of a Bethe lattice is seen on periodic lattices as well. For example, an infinite avalanche does not occur on any periodic lattice with $z = 3$ irrespective of the dimension d of the space in which the lattice is embedded [11].

One way to gain insight into the effect of z on σ_c is to study the model on lattices whose average coordination number z_{av} varies continuously between integer values. With this in mind infinite avalanches were studied on a dilute triangular lattice [14]. The study suggested that infinite avalanches occur when $z_{av} \geq 4$ but did not rule out a lower value. Next, the problem was studied on a Bethe lattice of a mixed coordination number such that a fraction c_4 of the sites had $z = 4$ and the remaining fraction $1 - c_4$ had $z = 3$. An exact solution was obtained and verified by numerical simulations. The result turned out to be somewhat surprising. Infinite avalanches can occur in the entire range $0 < c_4 \leq 1$ if $\sigma < \sigma_c$ where $\sigma_c \rightarrow 0$ continuously as $c_4 \rightarrow 0$ [15]. This suggests that the presence of a small fraction of $z = 4$ sites suffices to produce an infinite avalanche. However, the physical reason for this is not clear.

III. CRITICALITY ON A DILUTE $z = 4$ BETHE LATTICE

A dilute $z = 4$ Bethe lattice with only a fraction c of sites occupied by spins provides another example of interest that can be solved analytically. On such a lattice, there are sites with coordination numbers $z = 0, 1, 2, 3, 4$. This problem was studied earlier in the limit $c \rightarrow 1$, and $c \rightarrow 0$ to show how a tiny fraction of magnetic grains in geological rocks transforms familiar hysteresis loops into wasp-waisted loops [16]. In the following, we revisit this problem in the regime of moderate c to examine the dependence of σ_c on c . A key quantity is the conditional probability $Q^*(h, \sigma)$ that a nearest neighbor of an occupied site in the deep interior of a Cayley tree (the central site) is down at h before the central site is relaxed. The dynamics of the model is abelian i.e. the same final state is reached irrespective of the order in which the sites are relaxed. We start with all spins down on a Cayley tree and relax them in the following order: first we relax spins on the surface, then move towards the center relaxing spins on one level at a time. This amounts to calculating $Q^n(h, \sigma)$ for increasing n where $Q^n(h, \sigma)$ is the probability that a site on level n is down before its neighbor at level $n + 1$ is relaxed. Let us take the surface to be at level 0. A spin on the surface experiences a quenched random field h_i , an external field h , and a field $-J$ from the unrelaxed neighbor at level 1. When relaxed, it may flip up or stay down depending on the value of the net field $h_i + h - J$ on it. The probability $Q^0(h, \sigma)$ that it stays down is the probability that $h_i + h - J \leq 0$. Taking into account the Gaussian distribution of h_i , we get $Q^0(h, \sigma) = 0.5[1 + \text{erf}\{(J - h)/\sqrt{2\sigma^2}\}]$. An equation for $Q^n(h, \sigma)$, $n > 1$, is obtained similarly if we keep in mind that each spin to be relaxed at level n has one unrelaxed neighbor at level $n + 1$ and $z - 1$ relaxed neighbors at level $n - 1$. We get,

$$Q^n(h, \sigma) = \sum_{m=0}^{z-1} [Q^{n-1}(h, \sigma)]^m [1 - Q^{n-1}(h, \sigma)]^{z-1-m} q_{z,m+1}(h, \sigma) \quad (1)$$

Here $q_{z,m}(h, \sigma)$ is the probability that a z -coordinated spin with m neighbors down is down at applied field h .

$$q_{z,m}(h, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{(2m-z)J-h} e^{-\frac{h_i^2}{2\sigma^2}} dh_i = \frac{1}{2} \left[1 + \operatorname{erf} \left\{ \frac{(2m-z)J-h}{\sqrt{2\sigma^2}} \right\} \right]; \quad (m = 0, \dots, z)$$

The fixed-point $Q^*(h, \sigma)$ is given by, $Q^*(h, \sigma) = \lim_{n \rightarrow \infty} Q^n(h, \sigma)$. We find $Q^*(h, \sigma) > 1/2$ if $h < J$, $Q^*(h, \sigma) < 1/2$ if $h > J$. There is a discontinuity in $Q^*(h, \sigma)$ at $h = J$ and therefore an infinite avalanche in the system if $\sigma < \sigma_c$. The discontinuity decreases in size with increasing σ and vanishes at $\sigma = \sigma_c$. At σ_c , the two solutions at $h = J$ merge into $Q^*(J, \sigma) = 1/2$. The equation determining σ_c is,

$$A(A + 4B) = 0.$$

where

$$A = c^3 \{1 + q_{4,4}(J, \sigma) - 3q_{2,2}(J, \sigma)\} \text{ and } B = 1 - \{c^3 q_{4,4}(J, \sigma) + 3c^2(1-c)q_{3,3}(J, \sigma) + 3c(1-c)^2 q_{2,2}(J, \sigma) + (1-c)^3 q_{1,1}(J, \sigma)\}$$

The factor A is negative for all values of σ of interest. Therefore σ_c is effectively determined by the equation $A + 4B = 0$. No real positive value of σ_c satisfies this equation if c is less than a critical value c_{min} . Numerically, $c_{min} \approx 0.557$. The exact value (argument to be presented below) is $c_{min} = 2^{1/3}/(1 + 2^{1/3}) \approx 0.5575$. For $c > c_{min}$, σ_c increases continuously with increasing c starting from $\sigma_c = 0$ at $c = c_{min}$. The increase is remarkably steep in a narrow region adjacent to c_{min} . We may designate this region as the critical region. The width of the critical region is very small but the increase of σ_c in this region is substantial. Thus a plot of σ_c vs. c appears almost vertical at $c = c_{min}$. Theoretically, the slope of σ_c vs. c curve is infinite at c_{min} . It gradually decreases as one moves away from c_{min} but remains very large over the entire critical region. Figure (1) shows a plot of σ_c vs. c ; σ_c appears to rise vertically from $\sigma_c = 0$ to $\sigma_c \approx 0.275$ i.e. all values in the range $0 < \sigma_c < 0.275$ satisfy the equation at $c \approx 0.557$. Thereafter σ_c increases more gradually. At $c = 1$ we recover the known result $\sigma_c = 1.781$ for the undiluted $z = 4$ Bethe lattice. The data plotted in figure (1) was obtained using a standard numerical recipe for evaluating error functions in the expression for σ_c . The error in using this recipe is $\epsilon \leq 10^{-7}$. Within this error, σ_c at c_{min} rises vertically as shown in figure (1). However, if one uses another tool (Mathematica) to calculate the error functions with a greater precision ($\epsilon \leq 10^{-16}$), the vertical portion of the σ_c vs. c curve is replaced by a curve that bends slightly to the right in the narrow critical region; σ_c increases continuously from 0.0 to 0.307275 as c increases from $c_{min} \approx 0.5575$ to $c = 0.558$ [17]. This continuous but sharp increase in a narrow region would also appear to be nearly vertical when plotted on the scale of figure (1). The important point is that mathematical tools with higher precision as well as theoretical analysis agree that σ_c is a continuous, monotonic, but very steeply increasing function of c immediately above the threshold $c = c_{min}$. The situation brings to mind some (not so well understood) transitions in liquid crystals where the order parameter appears to jump discontinuously as in a first-order transition but the entropy changes continuously as in a second-order transition. In order to confirm this sharp change at $c \approx 0.557$, we performed simulations for $m(h, \sigma)$ at $\sigma = 0.4$ for $c = 0.55$ and $c = 0.57$. Theory predicts a discontinuity in $m(h, \sigma = 0.4)$ for $c = 0.57$ but no discontinuity for $c = 0.55$. This is what we observed in the simulations. Figure (2) shows the closeness between the numerical and corresponding theoretical results. The numerical results are obtained on a random graph rather than a Cayley tree in order to eliminate large surface effects. The initial state of the random graph is taken as all spins down. These simulations match the theoretical result on a Cayley tree if the surface spins are kept down, rather than relaxed at h . This procedure does not alter σ_c , but shifts the discontinuity from $h = J$ to $h > J$. The discontinuity moves closer to $h = J$ as σ increases, and vanishes at $h = J$ as $\sigma \rightarrow \sigma_c$. If the surface of the Cayley tree is relaxed at h rather than held in a fixed state, then the discontinuity occurs at $h = J$ only.

IV. GENERAL CRITERIA FOR INFINITE AVALANCHES

To recapitulate, (i) infinite avalanches occur if $z \geq 4$ and $\sigma < \sigma_c(z)$ but do not occur if $z = 2, 3$ (ii) on a lattice with $c : 1 - c$ mixture of $z = 4$ and $z = 3$ sites, infinite avalanches occur for all c ($0 < c \leq 1$) if $\sigma < \sigma_c(c)$, (iii) on a $z = 4$ lattice with a fraction c of sites occupied, infinite avalanches occur if $c > 0.557$ and $\sigma < \sigma_c(c)$. The reason

why infinite avalanches do not occur for large σ irrespective of other considerations is simple. Spins tend to flip up independently in the presence of large disorder, hence no infinite avalanche. Results (ii) and (iii) are puzzling at first sight; (ii) suggests that an arbitrarily small fraction of $z = 4$ sites is sufficient to cause an infinite avalanche but (iii) contradicts it because nearly 5% sites have $z = 4$ at $c \approx 0.557$. This requires further discussion. First, we look at the reason for (i). Absence of an infinite avalanche means $Q^*(h, \sigma)$ is continuous at $h = J$. It is easy to verify that $Q^*(J, \sigma) = 1/2$ is a fixed point irrespective of z and σ . The absence or presence of an infinite avalanche depends on the stability of this fixed point. If $Q^*(J, \sigma) = 1/2$ is stable, there is no discontinuity in $Q^*(h, \sigma)$ at $h = J$. An unstable $Q^*(J, \sigma) = 1/2$ splits into two stable fixed points, one larger and the other smaller than $1/2$ at $h = J$. Consequently the system jumps from a small magnetization state to a large magnetization via an infinite avalanche. The stability of $Q^*(J, \sigma) = 1/2$ is examined by turning down a small fraction of up sites on the surface of the Cayley tree and examining its effect on the next layer of sites. In other words, we increase the fraction of down sites on the surface from $1/2$ to $1/2 + \delta Q^0$, and calculate the fraction $1/2 + \delta Q^1$ of down sites on the layer next to the surface. Focus on a set of $z - 1$ sites on the surface which have a common neighbor, say B at the higher level. Consider the case when at least one of the $z - 1$ sites, say A is up and B is also up. Now if A is turned down, the local field at B gets reduced by $2J$. The probability that B will turn down as a result of it is given by $\delta Q^1(J, \sigma) = B_z \delta Q^0(J, \sigma)$. Using $q_{z,k} = q_{z,k}(h, \sigma)$ defined earlier, we obtain

$$B_z = (z - 1) \frac{1}{2^{z-2}} \sum_{m=0}^{z-2} \binom{z-2}{m} (q_{z,m+2} - q_{z,m+1})$$

Above equation is understood as follows: site A can be chosen in $z - 1$ ways, remaining $z - 2$ sites are down with probability $\frac{1}{2}$, $(q_{z,m+2} - q_{z,m+1})$ is the probability that site B is up if $m + 1$ of its neighbors are down but flips down if $m + 2$ neighbors are down, $q_{z,k}$ is the probability that a z -coordinated spin is down if k of its neighbors are down. The quantities $B_2 = q_{2,2} - q_{2,1}$, $B_3 = q_{3,3} - q_{3,1}$, and $B_4 = q_{4,4} + q_{4,3} - q_{4,2} - q_{4,1}$ are of special interest. B_2 and B_3 are less than unity for $\sigma > 0$; as $\sigma \rightarrow 0$, $B_2 \rightarrow 1$ and also $B_3 \rightarrow 1$. Hence the fixed point $Q^*(J, \sigma) = 1/2$ is stable and the possibility of an infinite avalanche is ruled out on a $z = 2$ or a $z = 3$ lattice. At $h = J$, B_4 simplifies to $B_4 = \frac{3}{2}(q_{4,4} - q_{4,2})$; $B_4 \rightarrow 3/2$ as $\sigma \rightarrow 0$. It decreases continuously with increasing σ ; $B_4 \rightarrow 1$ as $\sigma \rightarrow \sigma_c \approx 1.781$. Thus $Q^*(J, \sigma) = 1/2$ is unstable on a $z = 4$ lattice if $\sigma < \sigma_c$ and consequently there is an infinite avalanche in this case. This also confirms that σ_c obtained from considering the stability of $Q^*(J, \sigma) = 1/2$ is the same as obtained from requiring two roots of the fixed point equation to merge into each other. Figure (3) shows the initial value $Q^0(h, \sigma = J)$ and corresponding fixed-point value $Q^*(h, \sigma = J)$ in the neighborhood of $h = J$ for $z = 2, 3$, and 4 . For $h < J$, the bottom line represents surface $Q^0(h, J)$; higher curves show fixed points $Q^*(h, J)$ for $z = 2, 3$, and 4 respectively. The relative position of $Q^0(h, J)$ and $Q^*(h, J)$ gets reversed for $h > J$; $Q^*(h, J) > Q^0(h, J)$ if $h < J$, but $Q^*(h, J) < Q^0(h, J)$ if $h > J$. $Q^0(h, J)$ is of course continuous at $h = J$ and $Q^0(J, J) = 0.5$; $Q^*(h, J)$ is continuous at $h = J$ if $z \leq 3$ but discontinuous if $z = 4$. Figure (4) shows the growth for $z = 4$, and decay for $z = 2, 3$ of a small perturbation $\delta Q^0(h = J, \sigma = J)$ under successive iterations.

Next, we turn our attention to the dilute $z = 4$ lattice. In this case as well, $Q^*(J, \sigma) = 1/2$ is a fixed point. This fixed point must be unstable in the limit $\sigma \rightarrow 0$ if there is to be an infinite avalanche. In the limit $\sigma \rightarrow 0$, $B_2 \rightarrow 1$, $B_3 \rightarrow 1$, and $B_4 \rightarrow 3/2$. Consider a perturbation $\delta Q^0(J, 0)$ to the fixed point $Q^*(J, \sigma) = 1/2$. As we move from the surface of the tree towards its center, a $z = 4$ site increases the perturbation by a factor $3/2$, but $z = 3$ and $z = 2$ sites keep it unchanged. The $z = 1$ sites produce a new effect on the dilute lattice. They break the continuity of the path from the surface to the center. In our algorithm for relaxing sites, it is assumed that one of the neighbors of the site being relaxed, the one at a higher level, is present and unrelaxed. The bond with this neighbor ensures connection between adjacent levels of the tree. A $z = 1$ site breaks this connection with probability $\frac{3}{4}z_1$, where z_1 is the fraction of sites with one nearest neighbor only. If z_4 is the fraction of sites with 4 neighbors, then at every level of relaxation of the lattice, the perturbation is boosted with the probability $\frac{3}{2}z_4$, and terminated with probability $\frac{3}{4}z_1$. The critical point occurs when the two opposing effects balance each other, i.e. $z_1 = 2z_4$. Using $z_4 = c^5$, $z_1 = 4c^2(1 - c)^3$, the critical value of c is given by the equation $c^3 = 2(1 - c)^3$, or $c = 2^{1/3}/(1 + 2^{1/3}) \approx 0.5575$. The observed infinite avalanche on a mixed lattice with a fraction c_4 of $z = 4$ sites and $1 - c_4$ of $z = 3$ sites for $c_4 > 0$ is also understood in this light. The path from the surface to the center is never broken on the mixed lattice, and therefore an arbitrarily small presence of z_4 sites creates a gap in $Q^*(J, \sigma)$ in the deep interior of the tree.

V. CONCLUSION

To conclude, we have presented general criteria for the occurrence of infinite avalanches in the zero-temperature nonequilibrium random-field Ising model on a Bethe lattice. We find that infinite avalanches occur when all of the

following conditions are fulfilled: (i) σ is sufficiently small, (ii) there is a spanning cluster of occupied sites on the lattice, and (iii) the spanning cluster has a fraction of sites, even an arbitrarily small fraction, with connectivity $z \geq 4$. We have explained the reason for these conditions. The presence of an infinite avalanche on a mixed coordination lattice ($z = 3$ or 4) with an arbitrarily small fraction of $z = 4$ sites, and its absence on a dilute $z = 4$ lattice in a certain regime of dilution is now easily understood.

Our analysis also shows that disorder in the form of dilution of magnetic ions on a lattice affects hysteresis differently from disorder in the form of on-site random-fields. This is important because positional disorder in the form of vacancies is quite common in materials. We find a peculiar geometry driven transition near $c = 0.5575$ on a dilute $z = 4$ Bethe lattice. Similar behavior may be expected for $z > 4$ as well. Infinite avalanches vanish at this critical point continuously, but the slope of the continuous curve is nearly infinite. It appears as a first-order jump in the order parameter for all practical purposes. Bethe lattices often approximate real systems reasonably well. So this feature of the model may be observable in appropriate hysteresis experiments and useful in understanding other weakly first-order phase transitions as well.

Finally, we wish to end with a caution. We have made a case that a lower critical coordination number rather than a lower critical dimension determines critical hysteresis. Our suggestion is based on exact results on Bethe lattices and simulations on some periodic lattices. It conflicts with a widely accepted view in statistical physics community in favor of a lower critical dimension. Further work may be required to settle this issue but we mention two factors that may invalidate our suggestion. Results on Bethe lattices are essentially mean field results and do not necessarily have a bearing on criticality on periodic lattices in finite dimensions. Secondly, subtle corrections to scaling may explain the extant numerical results on periodic lattices without doing away with the importance of a lower critical dimension.

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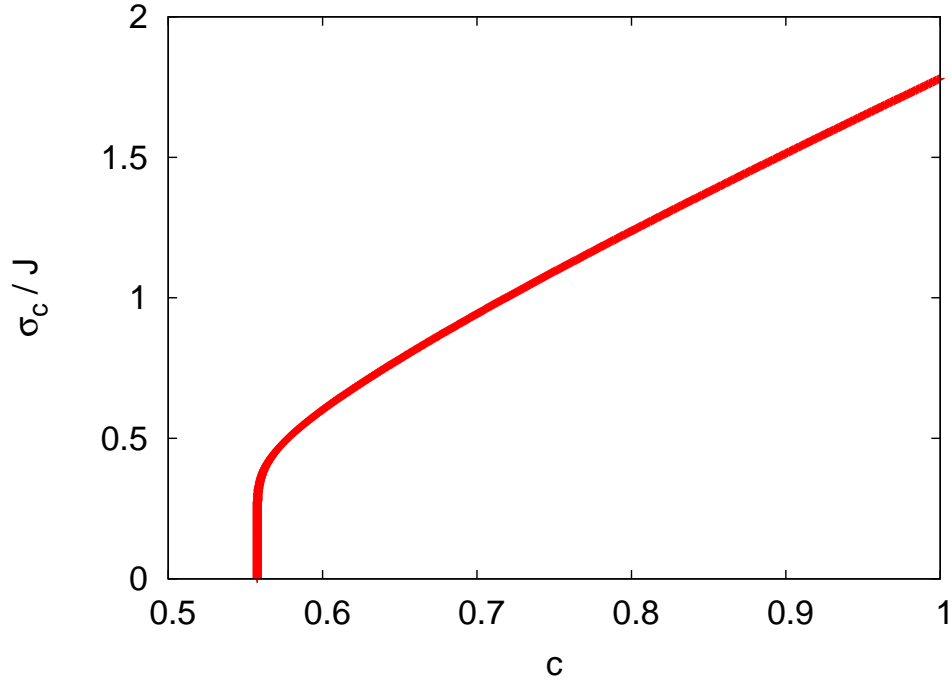


FIG. 1: Critical value of the standard deviation of the quenched random-field σ_c on a 4-coordinated random graph with a fractional occupancy c of its sites. The magnetization $m(h, \sigma)$ has a discontinuity if $\sigma < \sigma_c$. There is an almost vertical drop in σ_c at $c = 0.557$ approximately.

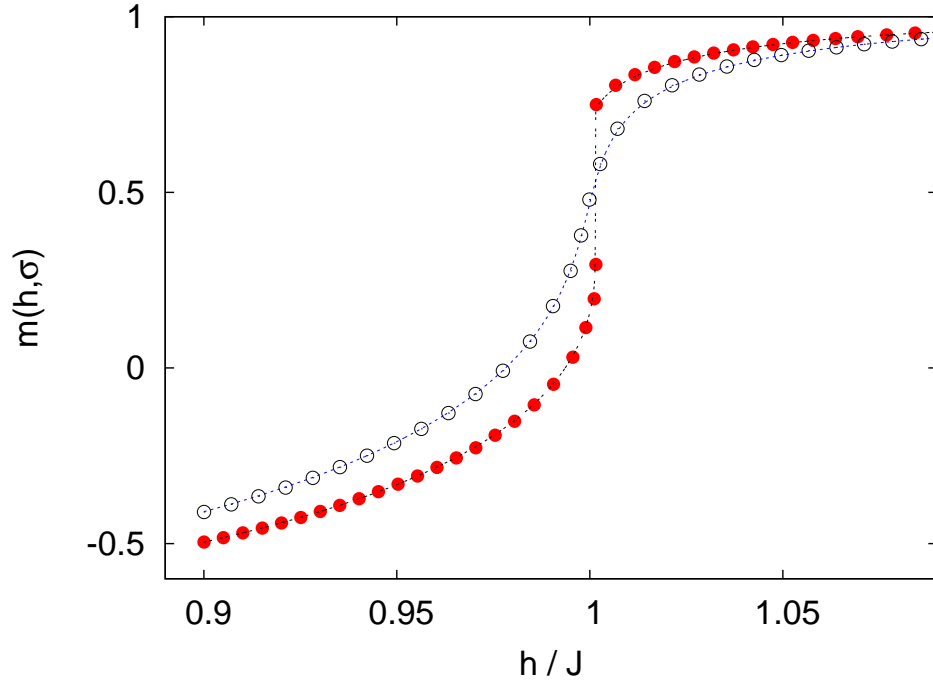


FIG. 2: Magnetization $m(h, \sigma)$ in increasing field h on a 4-coordinated dilute random graph for $c = 0.55$ (open circles) and $c = 0.57$ (filled circles) where c is the fraction of occupied sites on the graph. The quenched random field on occupied sites has mean value equal to zero, and standard deviation $\sigma = 0.40$. Theoretical predictions are superimposed on the respective simulations (a single run on $N = 10^7$ graph) and fit them quite well. The magnetization is smooth for $c = 0.55$ and has a jump for $c = 0.57$ as predicted by the theory.

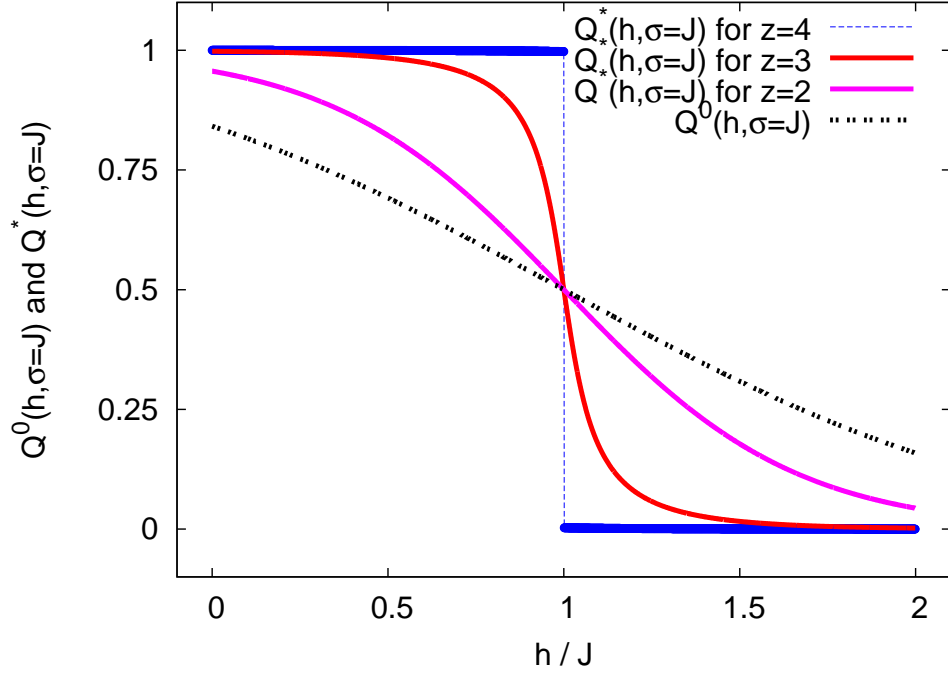


FIG. 3: The figure shows $Q^0(h, \sigma = J)$ and $Q^*(h, \sigma = J)$ in increasing field h (lower half of the hysteresis loop) on a Cayley tree of coordination number $z = 2, 3, 4$. The broken black line shows $Q^0(h, \sigma = J)$ which is common to all z because a surface site has only one neighbor irrespective of z ; $Q^0(h, \sigma = J)$ decreases continuously with increasing h and passes through the point $Q^0(h, \sigma = J) = 0.5$ at $h = J$. $Q^*(h, \sigma = J)$ for $z = 2$ (pink curve closest to the black broken line) and $z = 3$ (red curve next closest to the black broken line) behave similarly; both decrease continuously and pass through $Q^*(h, \sigma = J) = 0.5$ at $h = J$. However, as we go from $z = 2$ to $z = 3$, $Q^*(h, \sigma = J)$ becomes steeper at $h = J$, and generally moves farther away from $Q^0(h, \sigma = J)$. For $z \geq 4$, $Q^*(h, \sigma = J)$ acquires a discontinuity at $h = J$ as shown in the figure by the blue curve.

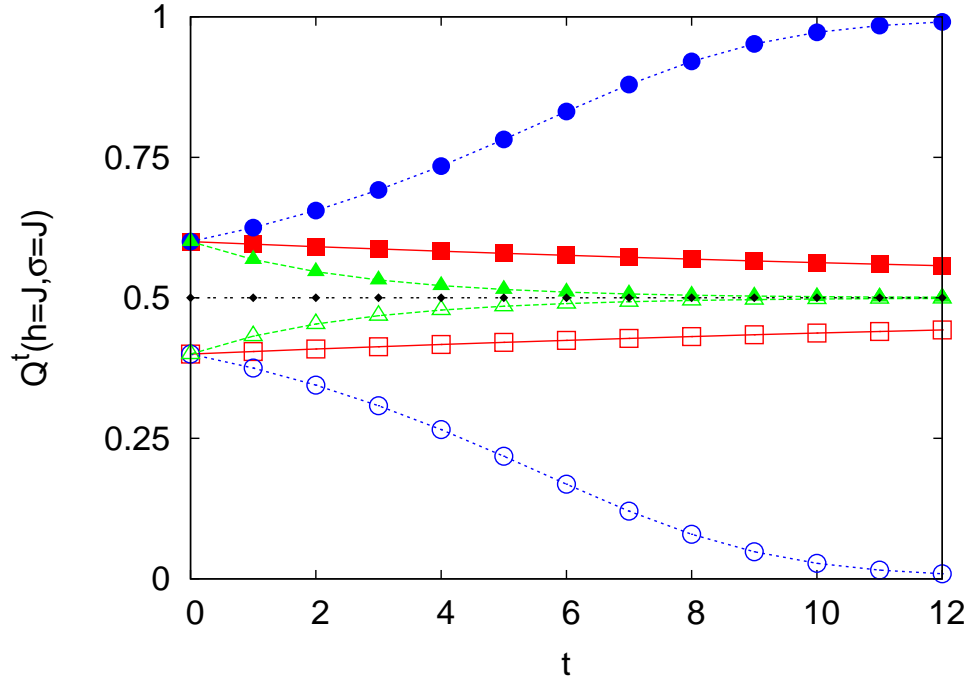


FIG. 4: The conditional probability $Q^t(J, J)$ (see text) at t successive levels of the Cayley tree starting from the surface ($t = 0$). If $Q^0(J, J) = 0.5$, $Q^t(J, J)$ remains equal to 0.5 as shown by the horizontal line. A small perturbation δQ^0 added to $Q^0(J, J)$ gradually decreases to zero if $Q^*(J, J) = 0.5$ is stable, but increases with t if $Q^*(J, J) = 0.5$ is unstable; perturbed $Q^t(J, J)$ lies on the same side of the horizontal line $Q^*(J, J) = 0.5$ as the initial perturbation δQ^0 . Figure shows $Q^*(J, J) = 0.5$ is stable for $z = 2$ (green triangles) and $z = 3$ (red squares), but unstable for $z = 4$ (blue circles). Filled symbols correspond to $\delta Q^0 > 0$ while the empty symbols correspond to $\delta Q^0 < 0$. The relaxed state on the first twelve levels of the tree is shown which suffices to make the trends clear.